

The Method 3 Computer Algorithm

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To produce my computer solutions for Method 3, I had to come up with an algorithm. I had to compute the adjacent interval content of **every** V-System chord. (The outer voice interval is simply the sum of the three adjacent voice intervals.) I spent a lot of time using pencil and paper, figuring and experimenting, searching for patterns. Finally I arrived at an aha! moment. Here is how the algorithm works:

We begin with one of the 43 four-note chord qualities and one of the 14 voicing groups. From that we will generate the adjacent interval content of four systematic inversions. That's the basic input and output. If we know how to do this correctly for one quality and one voicing group, we can apply the procedure to all of them.

The computer likes information in numerical form. So we express chord quality by the number of half steps between chord tones. We reference the voicing group in terms of its Method 2 chord tone gaps.

Let's say we want to find the interval content of the four systematic inversions of a V-4 maj7 chord. The maj7 quality, expressed in half steps, is: 1 - 4 - 3 - 4. The chord tone gaps for V-4 are: 2 1 0. These are our inputs.

When a chord tone gap is zero, we can fill it with one of the four intervals from the quality: 1 - 4 - 3 - 4. That is, we can fill it with a m2 (1 half step), a M3 (4 half steps), a m3 (3 half steps), or again a M3 (4 half steps).

When a chord tone gap is one, we can fill it with one of four "double sums." To calculate what I call "double sums," you take the quality, 1 - 4 - 3 - 4, sum each number to the right, and put the answer in the position from which you started summing. For maj7, the double sums are: 5 - 7 - 7 - 5. That is, (1 + 4) and (4 + 3) and (3 + 4) and (4 + 1). In other words, you begin with the quality a b c d where a, b, c, and d represent the number of half steps in the quality. Then the double sums are: (a + b) (b + c) (c + d) (d + a).

When a chord tone gap is two, we can fill it with one of the four triple sums, (a + b + c) (b + c + d) (c + d + a) (d + a + b). The triple sums for a maj7 chord are 8 - 11 - 8 - 9. Here's a table:

<u>Chord</u> <u>Tone</u> <u>Gap</u>	<u>Interval</u> <u>Fill</u>	<u>Formula</u>	<u>maj7</u> <u>Values</u>
0	single sums	a b c d (from the quality)	1 4 3 4
1	double sums	(a + b) (b + c) (c + d) (d + a)	5 7 7 5
2	triple sums	(a + b + c) (b + c + d) (c + d + a) (d + a + b)	8 11 8 9
3	(can't happen)		
4	single sums + 12	(a + 12) (b + 12) (c + 12) (d + 12)	13 16 15 16
5	double sums + 12	(a + b + 12) (b + c + 12) (c + d + 12) (d + a + 12)	17 19 19 17

The first part of the algorithm is calculating these sums. In our case, the low chord tone gap is 2 so we'll use triple sums for it. The mid chord tone gap is 1 so we'll use double sums. And the high chord tone gap is 0 so we'll use single sums, a.k.a. the half steps of the quality. Here's what we have:

high	1	4	3	4
mid	5	7	7	5
low	8	11	8	9

We almost have our interval content! The problem is alignment. We have to rotate the mid and high intervals to get proper alignment with the low intervals. The formulas I discovered for proper alignment are:

1. Rotate the mid row to the left by (1 + the low chord tone gap). (The 1 here is to account for the tenor chord tone.)
2. Rotate the high row to the left by (2 + the low chord tone gap + the mid chord tone gap). (The 2 here is to account for the tenor and alto chord tones.)

In our case, we have to rotate the mid row to the left 3 positions (1 + 2). We also have to rotate the high row to the left 5 positions (2 + 2 + 1). (Rotating one position to the left is equivalent to rotating five positions to the left. In computer-speak, you can always simplify the rotational shift by modulo 4.)

If we apply these two left rotational shifts, we have:

high	4	3	4	1
mid	5	5	7	7
low	8	11	8	9

We convert the number of half steps shown into human-friendly names for the intervals:

high	M3	m3	M3	m2
mid	P4	P4	P5	P5
low	m6	M7	m6	M6

and we have the interval content of the four systematic inversions for V-4 maj7. The row labeled "high" is the alto to soprano interval. The row labeled "mid" is the tenor to alto interval. The row labeled "low" is the bass to tenor interval. The columns are the four systematic inversions.

To recap, we begin with three rows of the quality in half steps. We sum them to the right, according to the chord tone gaps. Then we shift the upper two rows to the left, according to the formulas for proper alignment.

Explaining the algorithm is not so hard. But figuring it out in the first place was very difficult. At the same time, this kind of brain work is extremely fun. Who knew that the V-System held such interesting mental challenges?