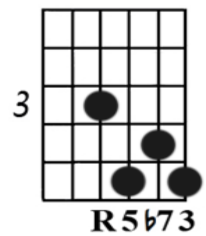
How Systematic Inversions Relate to the V-System

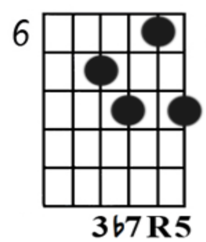
By James Hober

If you have come this far, you likely know what systematic inversions are. Nevertheless, to make sure we're all on the same page, I will quickly review how they work. The main focus of this chapter, however, is on how systematic inversions relate to Ted Greene's V-System.

Let's begin with an F7 chord:

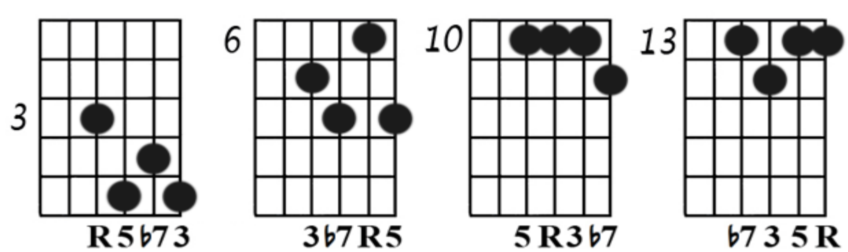


If you move the root in the bass up to the third, the fifth in the tenor up to the flat seventh, the flat seventh in the alto up to the root, and the third in the soprano up to the fifth, you arrive at the next inversion:



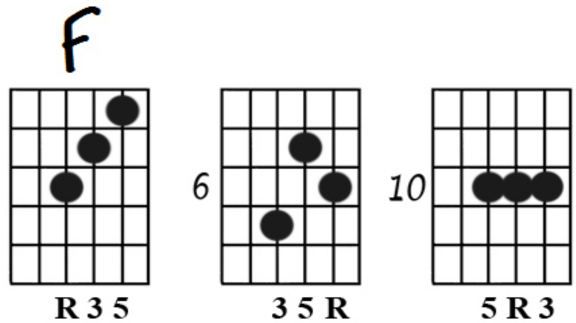
You move each chord tone to the next higher chord tone, usually keeping the move on the same string, and definitely keeping it in the same voice. The derived chord always remains in the same voicing group as the original chord. In this case, we started with a V-2 and therefore also finished with one.

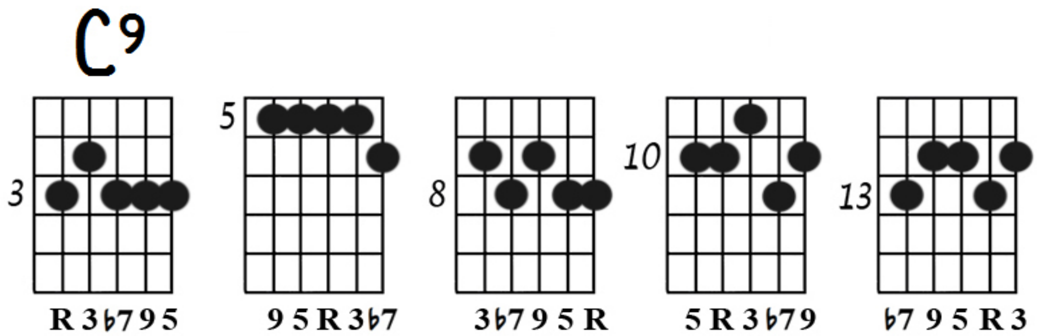
If we apply the same procedure to the new chord, we get another inversion, and another. In this way, we get a nice set of four chords, all in the same voicing group, that (usually) stay on the same set of strings. Here are the four systematic inversions of V-2 F7 on the top strings:

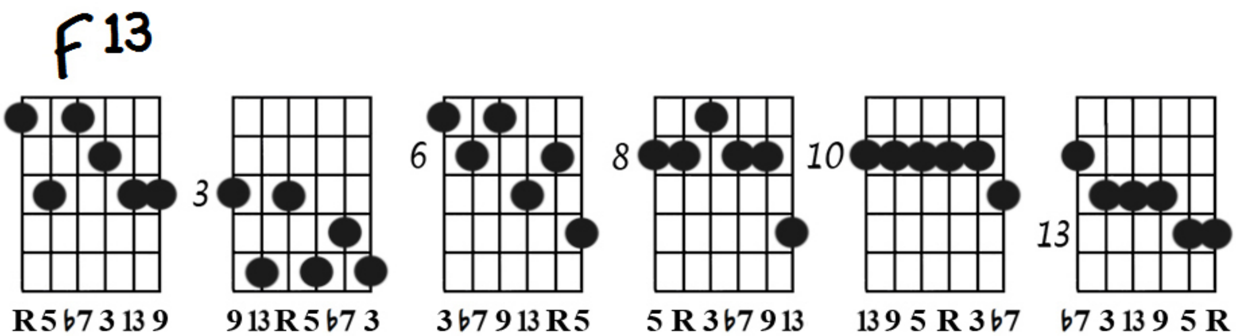


Less and More

If four-note chords can be systematically inverted, what about three-note, five-note, and six-note chords? Can they be systematically inverted? Sure. But, particularly with five- and six-note chords, some of the results may be difficult or impossible to finger and some may not sound good. Here are systematic inversions of an F major triad, a C9, and an F13:





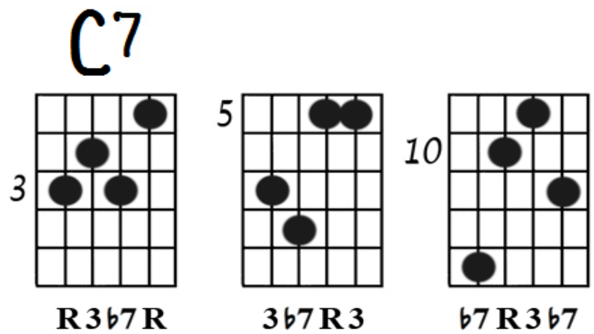


|----------possible to finger?--------|

As you can see, the number of distinct notes in a chord determines how many voicings there will be in a row of systematic inversions. For three-note chords, there are three systematic inversions, and so on. Ted was exploring other “V-Systems” for three-, five-, and six-note chords. I think he would have found success with a three-note chord V-System. (Perhaps in the future, someone will create an S-System with S-1, S-2, S-3, etc. for “spacing groups,” using S- to distinguish the three-note system from Ted’s four-note V-System.) For the bigger chords, Ted’s personal notes indicate that he was finding other, more advantageous ways of organizing them, such as grouping similar fingerboard shapes.

Double Trouble

What about doubling? The V-System is restricted to four-note chords without doubling. Does that mean that chords with doubling can't be systematically inverted? Let's try and see what happens:

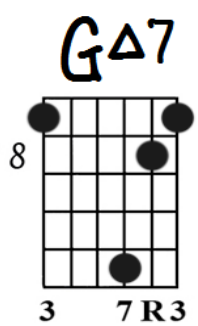


Because the initial C7 chord had only three distinct tones with the root doubled, systematic inversion yields only two more voicings, not three. The middle chord has a doubled third and sounds okay but it's nothing to write home about. The third chord with a doubled flat seventh sounds less convincing, in the conventional sense. So while it's possible to systematically invert chords with doubling, it often may not be fruitful.

The three V-System methods were designed with non-doubled chord types in mind. If you try to apply them to the chords above, the methods break down and are inconsistent. In Method 1, would the doubled C7 chords have Chronological Voice Formulas: [S and B together]TA, A[S and B together]T, TA[S and B together]? Would the Method 2 chord tone gaps be 0 0 0? But clearly these chords don't belong in V-1. And Method 3 says that V-1 has an outer voice interval of less than an octave. Here the outer voice interval **is** an octave. By restricting the V-System to non-doubled chord types, we avoid these inconsistencies and other problems.

Here's a fascinating excerpt from a Mark Levy lesson where Ted discusses trying to systematically invert chords with doubling:

<u>**Ted**</u>: [*plays*:]

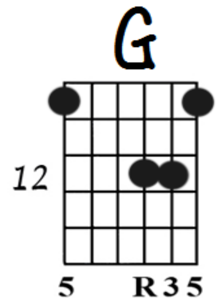


….which belongs to no voicing group because it's got two thirds, a root, and a seventh. These are incomplete chords, or doubled chords. This is our doubled friend 'cause it has two thirds. This is not an invertible chord. If you try to get the next G major seventh by moving each note up, three would go up to which tone?

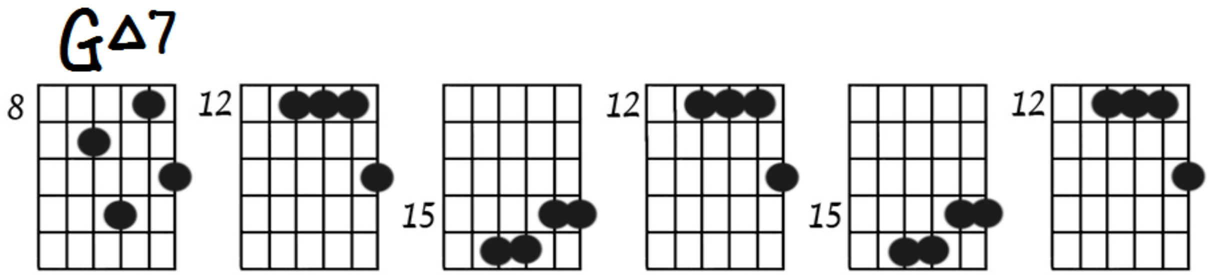
<u>**Mark**</u>: Five.
<u>**Ted**</u>: Seven would go up to what?
<u>**Mark**</u>: Root.
<u>**Ted**</u>: Root would go up to?
<u>**Mark**</u>: Three.
<u>**Ted**</u>: And three would go up to?
<u>**Mark**</u>: Five.

Ted: Good.

Now we'll have this:
No seventh around. It's nobody's fault.
It's just that when you have doubled voicings,
they don't produce the exact same chords as
you invert them.

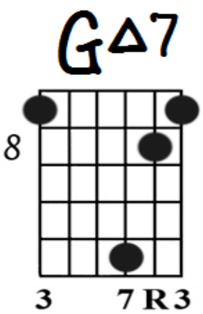


[*plays:*]

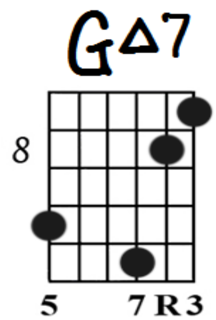


That's why we don't put them in a voicing group as such.

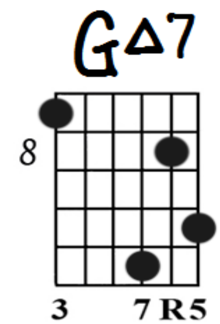
But this baby:



sure lives near V-6:



And it sure lives near V-7:



So I call it a hybrid, and there are going to be separate doubled groups between them when I publish the whole theory.

Mark: In the cracks.

Ted: Yeah, exactly.

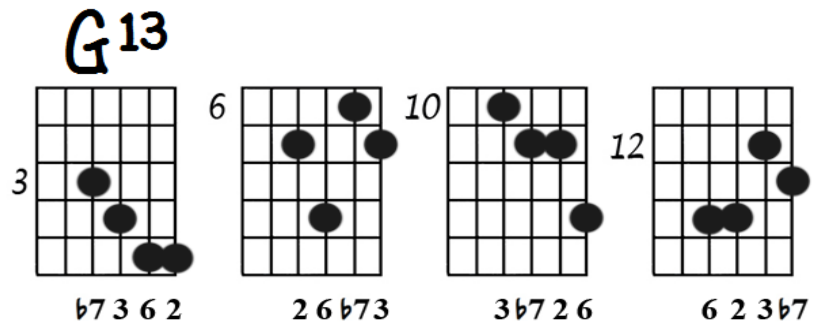
Mark: I hope you do.

Ted: Man, if the Creator keeps me here long enough I really intend to do this.

[*July 20, 1992, Mark Levy lesson at 14:15. Their guitars were tuned down about a half step.*]

Bring It Down

What about systematically inverting chords with ninths, elevenths, or thirteenths? When extensions are involved, we need to think of them as their lower octave equivalents: for 9 think 2, for 11 think 4, and for 13 think 6. This approach prevents the process of systematic inversion from straying into a different voicing group. Let's systematically invert a G13 no root, no fifth to illustrate this. If we arrange the chord tones in ascending order, 2 3 6 b7 (9 3 13 b7), we simply move to the right in the list to get the next higher chord tone:



The first and third chords of this set sound nice and are commonly used. They have the tritone between the 3 and b7 in the lower voices and the 9 and 13 extensions in the higher voices. The second and fourth chords are more dissonant and much less common. You can see how systematic inversion generates possibilities, but it's up to you to exercise taste and decide whether or not you want to use the newly derived voicings.

Since the V-System is an exploration of systematic inversions of every possible four-distinct-note chord, in (nearly) every reachable spacing, the same situation applies: you have to decide whether a voicing sounds good and is useful. Ted definitely was interested in extracting "choice" voicings to present to his students, and these can be found in his lesson sheets and personal notes.

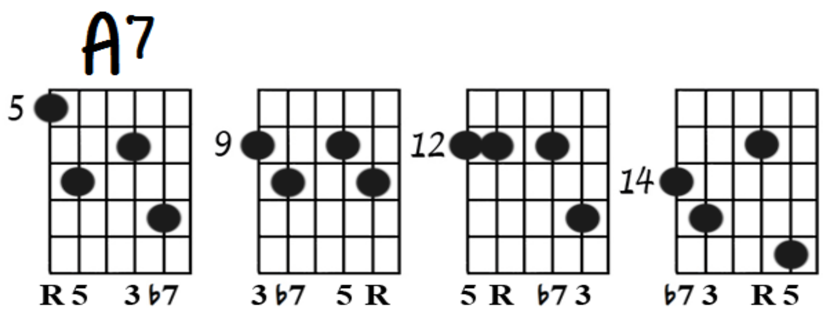
All three methods of the V-System use 2 for 9, 4 for 11, and 6 for 13. In *Method 1– How to Recognize*, *Method 1 – How to Build*, and *Method 2*, I stressed the importance of using the lower octave equivalents for extensions. The same principle applies to Method 3 but it is a little hidden. In *The Method 3 Computer Algorithm*, I stated that we begin with the number of half steps between chord tones for a quality. For a V-1 F/9, there are 2 half steps between the root and ninth (which is equivalent to the second), 2 between the ninth and third, 3 between the third and fifth, and 5 gets us back to the root: 2 - 2 - 3 - 5. By putting the chord in the tightest spacing (V-1) in order to calculate the half steps, we effectively are treating extensions as their lower octave equivalents. So all three methods require working with the lower octave equivalents just as systematic inversions do.

Now, let's examine how each method incorporates systematic inversions.

Method 1

In Ted's Method 1 Master Formula Table, each voicing group has four arrangements of the letters BTAS associated with it. For each voicing group, the four arrangements of the letters BTAS, a.k.a. the four Chronological Voice Formulas, relate to the four systematic inversions.

There are actually two ways to see this: hold the chord formula constant and rotate through the four Chronological Voice Formulas, or hold one Chronological Voice Formula constant and rotate the chord formula. Let's see how this works, for example, with a row of V-5 A7 systematic inversions:



First way:	R 3 5 b7	R 3 5 b7	R 3 5 b7	R 3 5 b7
	B A T S	S B A T	T S B A	A T S B
Second way:	R 3 5 b7	3 5 b7 R	5 b7 R 3	b7 R 3 5
	B A T S	B A T S	B A T S	B A T S

The first way, we hold the chord formula, R 3 5 b7, constant. Underneath it we write the four Chronological Voice Formulas for V-5. Notice that BATS, SBAT, TSBA, and ATSB are rotations of each other and are in order. That is, to get SBAT from BATS, we take the "S" on the end and rotate it around to the front. And so on. You can see that with each rotating Chronological Voice Formula lined up underneath the constant chord formula, it matches what's happening in the chord above it, in terms of chord tone placement.

The second way, we pick one of the Chronological Voice Formulas and keep it constant. We place the rotated chord formulas, in order, above the constant Chronological Voice Formula. Again, you can see that the alignment reflects what's happening in the chord above.

Using either the first way or the second way, we can generate the four systematic inversions. For the higher numbered voicing groups with an extra octave, you have an additional step: you simply insert the octave between the pair of voices specified in the Master Formula Table.

Be careful to avoid the following incorrect third way. If you look underneath the chord grids above, you see these chord tone orderings: R 5 3 b7, 3 b7 5 R, 5 R b7 3, and b7 3 R 5. Notice that you *do not* rotate the first one to get the second, and so on. To get the subsequent chord tone ordering, you systematically invert. That is, you move the root up to the third, the third up to the fifth, the fifth up to the flat seventh, and the flat seventh up to the root. But do not make the mistake of rotating these *chord tone orderings*. Doing so will take you into different voicing groups rather than generating systematic inversions in the same voicing group.

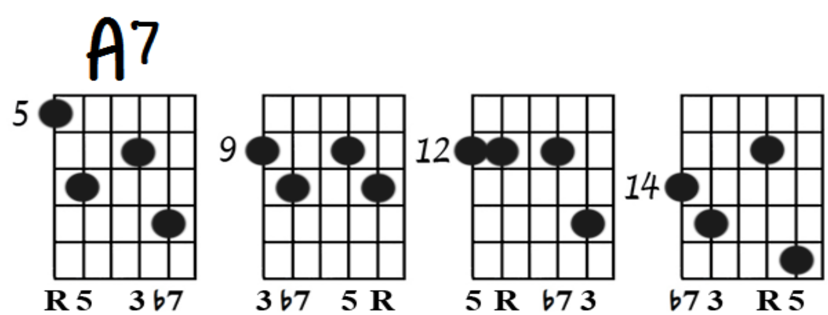
To summarize: the four Chronological Voice Formulas encapsulate the four systematic inversions when you hold the chord formula constant. Or, a single Chronological Voice Formula can be used to produce the four systematic inversions by rotating the chord formula.

We've looked at placing the four systematic inversions on a single set of strings. Of course, they often can be placed on more than one string set. No matter which strings are used, the above Method 1 relationships remain unchanged.

Method 2

The curious thing about Method 2, the Chord Tone Gap Method, is that the gaps do not change with systematic inversion. Method 2 expresses an invariant.

Let's look again at our example row of V-5 A7 systematic inversions:



The chord tone gaps in all these chords are the same. Between the bass and tenor you can insert one chord tone. Between the tenor and alto you can insert two. And between the alto and soprano you can insert one. A V-5 chord always has the chord tone gaps: 1 2 1.

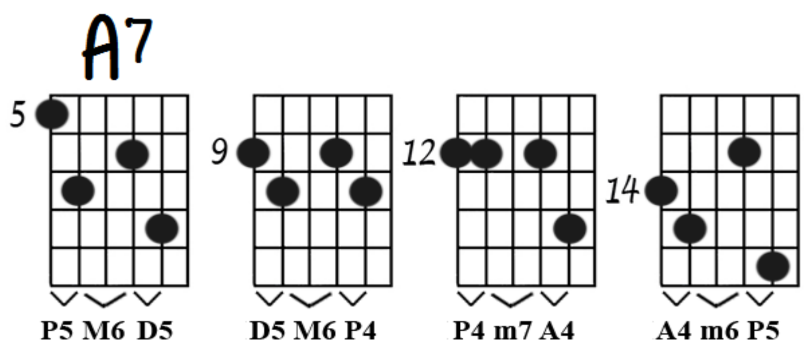
Systematically inverting a chord never changes the chord tone gaps. That simple fact is really all there is to say about how Method 2 relates to systematic inversions.

Method 3, Using Some Method 2

I already explained, in *The Method 3 Computer Algorithm*, how I calculated systematic inversion intervals for Method 3. But it's a little complicated so I'm going to go over it again here. This time I'm going to refer to the intervals the way musicians usually do: m2, M2, m3, M3, etc., rather than by the number of half steps they contain. ("M" stands for major, "m" for minor, "P" for perfect, "A" for augmented, and "D" for diminished.)

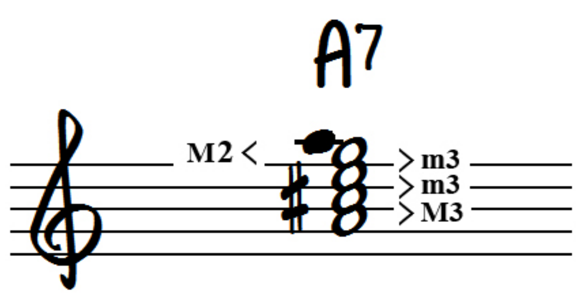
Remember, Method 3 is all about intervals: the outer voice interval and the three adjacent voice intervals between the bass and tenor, the tenor and alto, and the alto and soprano. Primarily we're going to concern ourselves with the adjacent voice intervals because once we have calculated those, it's a simple matter to add them together to get the outer voice interval.

Let's look again at the example V-5 A7 systematic inversions we have been using. This time, however, the adjacent voice intervals are shown underneath the grids, rather than the chord tones:



We need to figure out how to generate these intervals. In Method 3 terms, they describe the four systematic inversions of V-5 dominant seventh chords, regardless of the root note.

First, we need to define the dominant seventh chord quality. In its most compact form, it has the intervals: M3, m3, m3, M2. Let's call these our "basic intervals."



(The M2 here is slightly redundant, taking us from the flat seventh back to the root an octave higher, but by including it we can rotate the intervals for inversions.)

Second, we need to define V-5, and this is where I sneak a little Method 2 into Method 3. We're going to use the V-5 chord tone gaps: 1 2 1.

Since the chord tone gap between the bass and tenor is 1, we need to add together two of our basic intervals to fill this gap. That is, we need one of our basic intervals to go from the bass to the chord tone that could be inserted in the gap. Then we need another basic interval to go from the chord tone that could be inserted in the gap up to the tenor. So to calculate the four possible bass to tenor intervals, we add two neighboring basic intervals:

M3 + m3
m3 + m3
m3 + M2
M2 + M3

The results are: P5, D5, P4, A4. These results are the intervals we'll use between the bass and tenor in our systematic inversions.

Next, we have a chord tone gap of 2 between the tenor and alto. This means we must add three of the neighboring basic intervals together to fill this gap:

M3 + m3 + m3
m3 + m3 + M2
m3 + M2 + M3
M2 + M3 + m3

The results are: m7, m6, M6, M6. These results are the intervals we'll use between the tenor and alto in our systematic inversions.

Since the chord tone gap size of 1 between the alto and soprano is the same as the chord tone gap size between the bass and the tenor, we can re-use the lower voice intervals calculated earlier: P5, D5, P4, A4.

We have now gathered the following intervals:

Alto to Soprano:	P5	D5	P4	A4
Tenor to Alto:	m7	m6	M6	M6
Bass to Tenor:	P5	D5	P4	A4

We have all the correct intervals but they are not yet properly aligned. That is, column one above doesn't yet match the intervals in our root position V-5 A7, column two doesn't yet match the intervals in our first inversion V-5 A7, and so on. To fix this, we have to rotate the middle and top rows.

To align the middle row, we have to rotate it once to the left to account for the chord tone gap size of 1 between the bass and tenor. Then we have to rotate it once more to the left to account for the chord tone actually in the tenor. The order of the middle row needs to be:

Tenor to Alto:	M6	M6	m7	m6

To align the top row, we have to rotate it once to the left to account for the chord tone gap size of 1 between the bass and tenor, and then twice more to the left to account for the chord tone gap size of 2 between the tenor and alto. Then we have to rotate it twice more to the left to account for the chord tones actually in the tenor and alto. Altogether, we have to rotate it five times to the left. (Rotating once to the left is equivalent to rotating five times to the left.) The order of the top row needs to be:

Alto to Soprano:	D5	P4	A4	P5

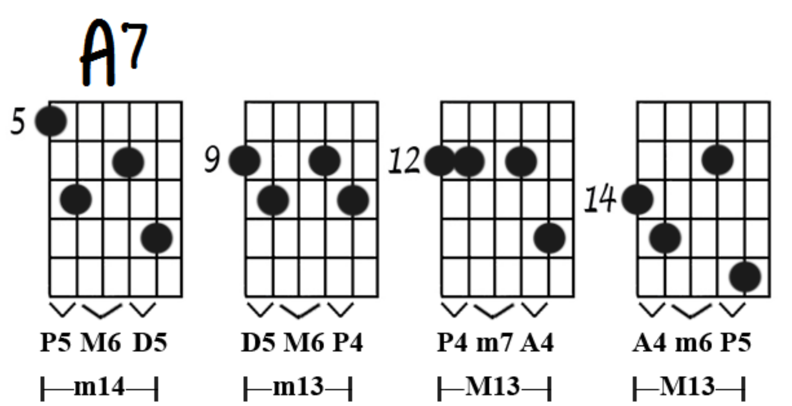
When we stack up our correctly ordered rows, we get the adjacent voice intervals in the V-5 A7 systematic inversions that we were aiming for:

	Root Pos.	1st Inv.	2nd Inv.	3rd Inv.
Alto to Soprano:	D5	P4	A4	P5
Tenor to Alto:	M6	M6	m7	m6
Bass to Tenor:	P5	D5	P4	A4

All that remains is summing of the adjacent voice intervals to get the outer voice intervals:

	Root Pos.	1st Inv.	2nd Inv.	3rd Inv.
Alto to Soprano:	D5	P4	A4	P5
Tenor to Alto:	M6	M6	m7	m6
Bass to Tenor:	P5	D5	P4	A4
Bass to Soprano:	m14	m13	M13	M13

This gives us the Method 3 interval content of the systematic inversions for V-5 A7:



To summarize: we begin with basic intervals of the quality in its most compact spacing. The adjacent voice intervals of the four systematic inversions are calculated by adding basic intervals as needed to fill the chord tone gaps. Then they are rotated to properly align. Finally, the adjacent voice intervals of each inversion are added together to get the outer voice intervals.

Method 3, Using Some Method 1

When I wrote the computer programs to complete Method 3, I used the above algorithm that makes use of Method 2 chord tone gaps to define each voicing group. Is it possible to instead use the Method 1 Master Formula Table to define each voicing group? In fact, it is. In retrospect, this may be considerably simpler.

As before, we define the dominant seventh quality using its basic intervals: M3, m3, m3, M2. These intervals are found between the chord tones as follows:

	M3		m3		m3		M2	
R		3		5		b7		R

This time, we define V-5 by its Method 1 Master Formula Table entry: BATS, SBAT, TSBA, ATSB. We apply the ascending chord formula, R 3 5 b7, to the four Chronological Voice Formulas, to get the following bottom up chord tone orderings:

B	T	A	S
R	5	3	b7
3	b7	5	R
5	R	b7	3
b7	3	R	5

Then for each of these four systematic inversions, we simply calculate the intervals between the chord tones. You can see how these intervals are sums of the basic intervals. For example, the interval between chord tones 5 and 3 (M6) is the sum of the basic intervals between 5 and b7, b7 and R, and R and 3 (m3+M2+M3):

R		**5**		**3**		**b7**
	P5 M3+m3		M6 m3+M2+M3		D5 m3+m3	
3		**b7**		**5**		**R**
	D5 m3+m3		M6 M2+M3+m3		P4 m3+M2	
5		**R**		**b7**		**3**
	P4 m3+M2		m7 M3+m3+m3		A4 M2+M3	
b7		**3**		**R**		**5**
	A4 M2+M3		m6 m3+m3+M2		P5 M3+m3	

By making use of the Method 1 Chronological Voice Formulas, we have derived the same adjacent voice intervals that we did before using chord tone gaps. We've also seen how these intervals, as before, are sums of the basic intervals.

Method 3 Revised to Stand on Its Own

Why was it necessary to use Method 2 or Method 1 to define the voicing groups for Method 3? Can't Method 3 stand on its own? The problem is that Ted's original Method 3 table won't always resolve a V-System chord to a single voicing group. In other words, Ted's Method 3 table, specifying the ranges of intervals for each voicing group, doesn't uniquely define each voicing group. To fix this, I now present a new, revised Method 3 table!

We've been referring to the basic intervals of the dominant seventh quality: M3, m3, m3, M2. In the general case, for **any** of the 43 qualities, we can call the basic intervals a, b, c, d.

In the specific case, where a is the interval between the root and the third, b is the interval between the third and the fifth, c is the interval between the fifth and the seventh, and d is the interval between the seventh back to the root, chord #1 below will be in root position, chord #2 in first inversion, chord #3 in second inversion, and chord #4 in third inversion. But in the general case, there may not be a root, third, fifth, and/or seventh. Hence, we refer to them simply as chords #1, #2, #3, and #4.

Here are the new Method 3 definitions of the fourteen voicing groups. The rows are the adjacent voice intervals. The columns are the four systematic inversions. The letters a, b, c, d are the basic intervals that define a quality:

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	c	d	a	b
V-1 =	T-A	b	c	d	a
	B-T	a	b	c	d

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a	a+b	b+c	c+d
V-2 =	T-A	c	d	a	b
	B-T	a+b	b+c	c+d	d+a

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a+b	a+b+c	b+c+d	c+d+a
V-3 =	T-A	b+c	c+d	d+a	a+b
	B-T	a	b	c	d

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	b	c	d	a
V-4 =	T-A	d+a	a+b	b+c	c+d
	B-T	a+b+c	b+c+d	c+d+a	d+a+b

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	b+c	c+d	d+a	a+b
V-5 =	T-A	c+d+a	d+a+b	a+b+c	b+c+d
	B-T	a+b	b+c	c+d	d+a

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	c	d	a	b
V-6 =	T-A	b	c	d	a
	B-T	a+8ve	b+8ve	c+8ve	d+8ve

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a	a+b	b+c	c+d
V-7 =	T-A	c	d	a	b
	B-T	a+b+8ve	b+c+8ve	c+d+8ve	d+a+8ve

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	c+d+a	d+a+b	a+b+c	b+c+d
V-8 =	T-A	d+a+b	a+b+c	b+c+d	c+d+a
	B-T	a+b+c	b+c+d	c+d+a	d+a+b

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a+8ve	a+b+8ve	b+c+8ve	c+d+8ve
V-9 =	T-A	c	d	a	b
	B-T	a+b	b+c	c+d	d+a

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a	a+b	b+c	c+d
V-10 =	T-A	c+8ve	d+8ve	a+8ve	b+8ve
	B-T	a+b	b+c	c+d	d+a

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	b+8ve	c+8ve	d+8ve	a+8ve
V-11 =	T-A	d+a	a+b	b+c	c+d
	B-T	a+b+c	b+c+d	c+d+a	d+a+b

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	d+a+b	a+b+c	b+c+d	c+d+a
V-12 =	T-A	b+c	c+d	d+a	a+b
	B-T	a+8ve	b+8ve	c+8ve	d+8ve

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	c	d	a	b
V-13 =	T-A	b+8ve	c+8ve	d+8ve	a+8ve
	B-T	a	b	c	d

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	c+8ve	d+8ve	a+8ve	b+8ve
V-14 =	T-A	b	c	d	a
	B-T	a	b	c	d

(Each column above can be summed to get the outer voice interval.)

The new table above dramatically simplifies Method 3. It precisely expresses the relationship between the four systematic inversions and their adjacent voice interval content. It makes building V-System chords using Method 3 a snap. Recognizing chords is also straightforward: just find the basic intervals for the quality and see if each adjacent voice interval in the chord is a basic interval (a, b, c, d), double sum (a+b, b+c, c+d, d+a), triple sum (a+b+c, b+c+d, c+d+a, d+a+b), or one of those + an octave. With that info, the revised Method 3 table will tell you the voicing group. The new, revised Method 3 table has no dependency on Method 1 or Method 2, other than the fact that all the methods are interrelated at their core. I certainly would have included it in my Method 3 explanation chapters had I worked it out before now!

Deriving Ted's Original Method 3 Table from the Revised Method 3 Table

The largest value that a, b, c, or d can take in the revised Method 3 table above is M6. This largest basic interval can be found only in the most dissonant of the 43 qualities: 1 - 1 - 1 - 9. (The 9 half steps are the M6 interval.) The smallest value a, b, c, or d can take is a m2. So if we put the values m2, m2, m2, M6 into a, b, c, d above, we get the systematic inversions of the most dissonant quality. This, in turn, gives us the ranges of possible adjacent voice intervals, the extreme limits, for each voicing group. We can then sum the columns of adjacent voice intervals to get the range of possible outer voice intervals for each voicing group. So by plugging in the basic intervals of the most dissonant quality (m2, m2, m2, M6) into the revised Method 3 table, we can derive Ted's original Method 3 table, which shows the ranges of adjacent voice and outer voice intervals for each voicing group.

To illustrate, let's calculate the interval ranges for one voicing group. For example, take V-4. The table shows:

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	b	c	d	a
V-4 =	T-A	d+a	a+b	b+c	c+d
	B-T	a+b+c	b+c+d	c+d+a	d+a+b

We set a = m2, b = m2, c = m2, and d = M6 and get:

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	m2	m2	M6	m2
V-4 =	T-A	M6+m2	m2+m2	m2+m2	m2+M6
	B-T	m2+m2+m2	m2+m2+M6	m2+M6+m2	M6+m2+m2

We sum the intervals and get:

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	m2	m2	M6	m2
V-4 =	T-A	m7	M2	M2	m7
	B-T	m3	M7	M7	M7

This gives us the ranges of adjacent voice intervals for V-4:

		Smallest	Largest
	A-S	m2	M6
V-4 =	T-A	M2	m7
	B-T	m3	M7

Since we worked with the most extreme quality, we found the adjacent voice interval limits for all V-4. Now we sum the columns to get the outer voice interval limits:

		Chord #1	Chord #2	Chord #3	Chord #4
	A-S	m2	m2	M6	m2
V-4 =	T-A	m7	M2	M2	m7
	B-T	m3	M7	M7	M7
sum:	B-S	M9	M9	m14	m14

And this gives us, for all V-4, the range of outer voice intervals: M9 to m14 (an octave + m7).

Ted's original Method 3 table expressed the V-4 ranges this way:

	S		
		m2 – M6	
	A		M9
V-4		M2 – m7	to
	T		b14 (b7)
		m3 – M7	
	B		

The revised Method 3 table (with a, b, c, and d) can be used to calculate the interval ranges in Ted's original Method 3 table. But it goes further in that it uniquely defines each voicing group.

Conclusion

Prior to inventing the V-System, Ted knew about systematic inversion. He created the V-System to organize four-note systematic inversions into voicing groups, based on their spacing. Each of the three methods is a different way to classify them into the fourteen voicing groups. Therefore, as we have seen, each of the three methods has a different relationship to systematic inversion. And yet at their core, all three methods share a deep affiliation.

– James