The 43 Four-Note Qualities

By James Hober

Ted Greene methodically worked out that there are 43 four-note qualities. He did this by laboriously writing out every possibility:

C Db D Eb
C Db D E
C Db D F
C Db D Gb
etc.

He wrote out such lists multiple times to check himself. Probably the last time he wrote out such a list was on 5-18-1985. Ted's original page (with a transcription) has been posted to tedgreene.com so you can examine it. You will see that Ted crossed out duplicates as he worked. To understand how he identified a duplicate, we have to clarify how he defined a four-note quality.

What Exactly Did Ted Mean by a Four-Note Quality?

If we remove the root from a chord's name, we have its quality. The quality of a Cm7 chord is m7. In addition to common chord qualities like m7, there are many unusual ones in the V-System, like Δ7#9 no 5. You don't encounter a Δ7#9 no 5 in your average polka. But because the V-System includes every possible four-distinct note chord quality, we count both common and uncommon qualities. When Ted listed the 43, he had in mind a definition of a quality that we can understand in terms of four restrictions.

The first restriction is that we take four different notes from the chromatic scale. If two of the notes are the same, we have a doubling and the V-System does not include chords with doubling. Chords with doubling will not produce systematic inversions with four different notes. They may be useful and sound good but we set them aside to investigate chords with four distinct notes. In counting the 43, we exclude any chords with doubling.

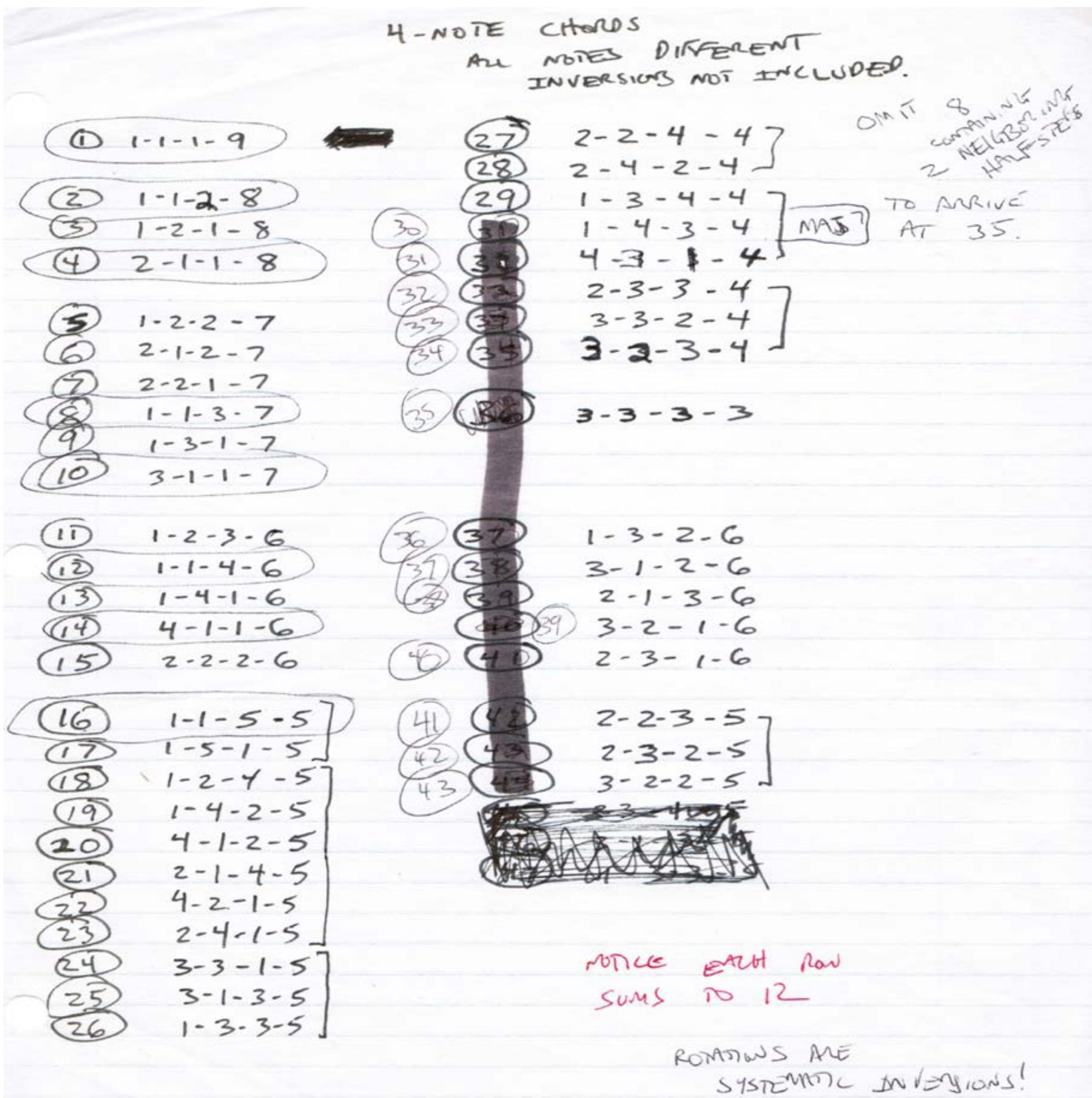
If I play "Twinkle, Twinkle, Little Star" for you in the key of C and then play it for you in the key of Bb, you will recognize it as the same tune. The notes are different but the relationships between the notes are the same. Similarly, if I transpose a four-note chord, I get another chord of the same quality. So when we count C7 as one of the 43, we can eliminate Db7, D7, Eb7, etc. Transposition does not change quality. Therefore the second restriction is that we exclude transpositions.

When you get right down to it, what really matters is the set of intervals between the notes. When we take a closely spaced C7 chord (in V-1 voicing), we find it has a major third between C and E, a minor third between E and G, and a minor third between G and Bb. This interval content uniquely defines a dominant seventh chord quality.

We Interrupt This Program for Ted's New Notation

By the late 1980s, Ted had worked out a new way of notating each quality. He expressed each adjacent voice interval by the number of half steps it contains. For C7, there are four half steps between C and E, three between E and G, three between G and Bb, and two to get from Bb back to C an octave higher. The dominant seventh quality can be represented in Ted's new notation like so: 4 - 3 - 3 - 2. The four numbers in this notation always sum to 12, the number of half steps in an octave.

Here's a copy of a page I kept from a lesson, where Ted wrote out all 43 qualities in his new notation. It took him a matter of minutes to write this out for me. Whereas, it took him hours to write out the 43 by note name, as he did on his 5-18-1985 personal sheet. Below, the dashed number combinations starting with "1 - 1 - 1 - 9" are in Ted's handwriting. The circled numbers counting them up to 43 are in my handwriting. (Notice I made a mistake and had to black out my miscount.) The comments at the top of the page are probably remarks Ted made in the lesson that I jotted down. The comments at the bottom of the page are realizations I later had back at home.



4-NOTE CHORDS
ALL NOTES DIFFERENT
INVERSIONS NOT INCLUDED.

1 1-1-1-9
2 1-1-2-8
3 1-2-1-8
4 2-1-1-8
5 1-2-2-7
6 2-1-2-7
7 2-2-1-7
8 1-1-3-7
9 1-3-1-7
10 3-1-1-7
11 1-2-3-6
12 1-1-4-6
13 1-4-1-6
14 4-1-1-6
15 2-2-2-6
16 1-1-5-5
17 1-5-1-5
18 1-2-4-5
19 1-4-2-5
20 4-1-2-5
21 2-1-4-5
22 4-2-1-5
23 2-4-1-5
24 3-3-1-5
25 3-1-3-5
26 1-3-3-5

27 2-2-4-4
28 2-4-2-4
29 1-3-4-4
30 31 1-4-3-4 MAJ7
31 32 4-3-1-4
32 33 2-3-3-4
33 34 3-3-2-4
34 35 3-2-3-4
35 36 3-3-3-3
36 37 1-3-2-6
37 38 3-1-2-6
38 39 2-1-3-6
40 39 3-2-1-6
40 41 2-3-1-6
41 42 2-2-3-5
42 43 2-3-2-5
43 43 3-2-2-5
[redacted]

OMIT 8 CONTAINING 2 NEIGBORING HALFSTEPS
TO ARRIVE AT 35.

NOTICE EACH ROW SUMS TO 12

ROTATIONS ARE SYSTEMATIC INVERSIONS!

And Now We Return to Our Regularly Scheduled Program

If we invert a C7, so that the notes from the bass up are E G Bb C, we haven't changed the quality. It's still a dominant seventh. Inversion does not change quality. In Ted's new notation, inversions are rotations:

4 - 3 - 3 - 2,
3 - 3 - 2 - 4,
3 - 2 - 4 - 3,
2 - 4 - 3 - 3

represent root position, first inversion, second inversion, and third inversion respectively of the dominant seventh chord. In counting the 43, we only count one of the four systematic inversions. It doesn't matter which one we count, as long as we don't count the others. The third restriction is that we count only one inversion per quality.

Homonyms are chords that sound alike but have different meanings. That is, they function differently in a chord progression. F6 and Dm7 are homonyms because they contain the same notes but are used differently. F6 is commonly used as a I chord. Dm7 is commonly used as a ii chord. In counting the 43, F6 and Dm7 are considered the same quality. They can both be represented as 2 - 3 - 4 - 3 in Ted's new notation. On his Seven Basic Qualities sheets, Ted treated the 6 and m7 chords as **different** qualities. But in counting 43 different four-note qualities, Ted considered them the **same** quality. The fourth restriction is that we count any possible homonyms as a single quality.

To summarize, in precisely defining a four-note chord V-System quality,
(1) doublings are not allowed,
(2) transpositions are excluded,
(3) only one of four inversions is counted,
(4) only one of multiple homonyms is counted.

So when Ted tallied the 43 qualities on 5-18-85, he worked systematically in order to avoid doublings. He crossed out duplicates that were transpositions and/or inversions of previous qualities. And he wrote some but not all of the homonyms he knew for each quality.

43 = 8 + 35

Ted divided the 43 qualities into two groups: the 8 very dissonant qualities and the 35 regular qualities.

All 8 very dissonant qualities contain two neighboring half steps in V-1 spacing. Ted called such neighboring half steps "chrom tones" in his personal notes, "chrom" being short for "chromatic." The most dissonant of the 8 contains three neighboring half steps. As we will see, this most dissonant cluster of half steps, 1 - 1 - 1 - 9, is important for Method 3, where it is used to discover the range of possible intervals for each voicing group.

Ted felt that dissonance was an acquired taste. A little child may be frightened by a harsh chord. Over time, with growing listening experience, the ear can become accustomed to, say, dominant chords with chromatic alterations. Also, context matters: When we watch a movie, we can accept dissonant music that might not appeal to us outside of that context. Also, broadly speaking, an important trend in music history has been the growing acceptance of greater and greater dissonance.

Nevertheless, the dissonant 8 qualities push the limits. If you're interested in creating music with strong dissonance, the dissonant 8 may be for you. Since Ted was mostly concerned with tonal music, including jazz and blues, the dissonant 8 were marginal for him, much less useful. Even among the 35 regular qualities, there are unusual chords, "many of which sound better or more effective if arpeggiated," Ted wrote in his personal notes.

The V-System grew out of Ted's commitment to mathematically generate every possible four-note chord. The personal page that he began on 4-18-80 shows him working out every reasonable (and sometimes unreasonable) homonym for each of the 43 qualities. At the bottom of this intense page, he tasks himself:

1. Systematically find all useable voicings of all these chords.
2. Find all systematic inversion rows (V-1 – V-14) of all these chords, in all fingerings.

That plan became his V-System. But the word "useable" is very important here. Ted's ear and musicality were the final arbiters for him. He did exhaustive systematic work. But his reward and delight were finding gems, wonderful sounding guitar chords that hadn't been discovered or used before. And sharing those discoveries.

Below are listed all 43 four-note qualities in Ted's new notation, specifying the number of half steps between adjacent voices in V-1 spacing. Following in Ted's footsteps, I have analyzed the 35 regular qualities from all 12 possible root notes to find what I consider to be sensible homonyms. I have checked the homonyms against Ted's 4-18-80 and 5-18-1985 personal pages. What I judge to be the most common name is listed first on the root C. Then follow increasingly less common homonyms for the chord.

Exhaustive Homonym Analysis and Where to Draw the Line

How did I decide what is a reasonable homonym? Consider the notes C, E, G, Bb. The most obvious name for a chord with those notes is C7. Another name for the chord is F#7b9b5 no R because, thinking from the root F#, C is the b5, E is the b7, G is the b9, and Bb (equivalent to A#) is the 3. There's no F# in the chord so we write "no R" for "no root."

We've found two names, or homonyms, for this chord. Are there more? To make sure that we don't miss any homonyms, we must consider the chord from all twelve possible roots and see if each name makes sense:

From Root	Chord Tones	Name
C	R 3 5 b7	C7
B	b9 4 #5 7	BΔ7b9sus+ no R, 3 ???
Bb	9 #11 6 R	Bb6/9/#11 no 3, 5
A	#9 5 b7 b9	A7#9b9 no R, 3
Ab	3 #5 7 9	AbΔ9+ no R ???
G	4 6 R b3	Gm6/11 no 5 ???
F#	b5 b7 b9 3	F#7b9b5 no R
F	5 7 9 4	FΔ9sus no R
E	#5 R #9 b5	E(7)#9b5+ no 3, b7
Eb	6 b9 3 5	Eb13b9 no R, b7
D	b7 9 11 #5	D11+ no R, 3 ???
Db	7 b3 #11 6	DbmΔ7/6/#11 no R, 5 ???

In my opinion, the names marked with "???" are pushing it too far. You may consider some of the other homonyms I accept as still too ridiculous. Or you may think homonyms I reject are fine. Where you draw the line is a judgment call.

It was a lot of work for me to analyze the 35 regular qualities from all 12 possible roots. Then I had to order them, placing what I judged to be the most common name first. Did Ted do such exhaustive analysis? You bet. His tattered, scribbled personal page, dated as having been begun on 4-18-80, analyzed all 35 regular qualities from all 12 roots. He rechecked his analysis 6-8-84. So evidently he returned to this page again and again over a number of years.

Why didn't I just use Ted's page? Why did I redo all the analysis myself? For one thing, Ted himself probably would have filtered out some of the homonyms he came up with. If you think my list goes too far, his included even more awkward names. Some of these he placed in parentheses, indicating that he understood they were stretches. A few times, he made mistakes. Therefore my analysis is an extra check on his work. I take responsibility for any mistakes that remain in the list. The main reason I didn't just transcribe Ted's scrawled page is that it's really hard to read. Tiny writing in different colors is layered on top of itself. Some of it is smudged, unreadable, and torn.

The main points where my naming differs from Ted's:

- I use #11 less than he did. If a dominant chord also contains 5, I use #11 in the name. Otherwise, I use b5.
- In addition to using + for chords with #5, Ted sometimes wrote b6 or b13. I never do. I only consider this tone as #5 and label the chord with a +.
- I indicate the m7b5 chord as such. Ted wrote ^{ø}7.

- A diminished triad or diminished seventh chord can have extensions. You can take notes of the diminished scale that are not part of the dim7 chord and add them to the chord: 9, 11, #5, and 7. Each diminished extension is a half step below a dim7 chord tone. All four diminished extensions comprise another dim7 chord located a half step below the original chord. Ted named such chords, for example, "C#°ext," where I write more specifically "C#°Δ7+ no R, b3." Sometimes Ted specified a chord as having a diminished extension with an added tone that is not part of the diminished scale. I do not.
- I edited down the possibilities more than he did on his 4-18-80 sheet.

If the complete list below is too overwhelming, just look at the first name or two or three.

The 43 Four-Note Chord Qualities

1) 1 - 1 - 1 - 9
2) 1 - 1 - 2 - 8
3) 1 - 1 - 3 - 7
4) 1 - 1 - 4 - 6
5) 1 - 1 - 5 - 5
6) 1 - 1 - 6 - 4
7) 1 - 1 - 7 - 3
8) 1 - 1 - 8 – 2

The eight very dissonant qualities containing two neighboring half steps.

1) 1 - 2 - 1 - 8 CmΔ9 no 5 = D13b9 no 3, 5 = B(7)#9b9 no 5 = F7/6/#11 no 1, 3 = Ab(7) #9#11 no R, b7 = Eb°Δ7+ no b3, b5 = F#°/11+ no R,b3 = A°/9/11 no R,6

2) 1 - 2 - 2 - 7 CΔ9 no 5 = D13 no 3, 5 = Am/9/11 no R = Ab(7)#9b5+ no R, b7 = FΔ7/6/#11 no R, 3

3) 1 - 2 - 3 - 6 C7/11 no R = Gm7/6 no 5 = FΔ9sus no 5 = Bb6#11 no 3 = E(7)#9b9b5 no 3,b7 = Db13#9b5 no R, b7 = Dm+/9/11 no R

4) 1 - 2 - 4 - 5 C7/6 no 5 = Gb7#9b5 no R = BbΔ9#11 no 3, 5 = Gm13 no R, 5, b7 = A(7)#9b9 no 3, b7 = FΔ7/11 no R

5) 1 - 2 - 5 - 4 CΔ9 no 3 = Am11 no R, 5 = D7/6sus no 5 = Gadd11 = F6/9/#11 no R, 3 = B(7)#9b9+ no 3, b7

6) 1 - 2 - 6 - 3 C(7)#9b5 no b7 = C° add♮3 = Gb7/6/b5 no 3 = B11b9 no R, b7 = C#mΔ9/11 no R, 5 = EΔ9+ no 3 = Bb°9+/11 no R,b3,6 = G°Δ7+/11 no R,b3,b5

7) 1 - 2 - 7 - 2 C7/6 no 3 = Gm/9/11 = BbΔ13 no 3, 5 = A7#9b9 no 3, 5 = F11 no R, b7 = Gb(7)#9b9b5 no R, b7 = Eb6#11 no R = E°+/11 no R, 6 = C#° Δ7+ no R, b3

8) 1 - 3 - 1 - 7 C11b9 no 5, b7 = Bbm/9/#11 no R, b7 = DbΔ7#9 no 5

9) 1 - 3 - 2 - 6 C7#11 no 3 = D11+ no R, 9 = A13#9b9 no R, 3, 5 = Gb(7)b9b5 no b7 = GmΔ7/11 no 5 = Eb(7)/6/#9 no R, b7 = E°9+ no R, 6 = Bb°9+ no b3, b5

10) 1 - 3 - 3 - 5 C7#9 no R = Gb13b9 no R, 5 = A7b9#11 no R, 3 = Eb(7)b9 no b7 = C#m6/9/#11 no R, 5 = EmΔ7b5 = E°Δ7 = G°7+ no b5 = Bb°7/11 no b3

11) 1 - 3 - 4 - 4 CmΔ7 = Am9b5 no R = D13b9sus no R, 5 = F9#11 no R, 3 = B(7)b9+ no b7

12) 1 - 3 - 5 - 3 C with 3 and b3 = Gb13b9b5 no R, 3 = Eb13b9 no 5, b7 = B11b9+ no R, b7 = Am7/#11 no R = EmΔ 7+ = Db°Δ9 no R, 6 = Bb°7/9/11 no R, b3 = G°+/11 no b3, b5

13) 1 - 3 - 6 - 2 C7b9 no 5 = Eb13b9 no R, 3 = Gb7#11 no R = A(7)#9b9 no R, b7 = Ab11+ no R, b7 = Bb°/9 = G°7/11 no R = E°7+ no b3 = C#m6Δ7 no 5

14) 1 - 4 - 1 - 6 CΔ7#11 no 3 = Am13 no R, 5 = D7/6/11 no R, 5 = Eb13#9+ no R, b7 = GΔ7/11 no 5

15) 1 - 4 - 2 - 5 C13 no R, 5 = Gm6/9 no R = BbΔ7#11 no 5 = Gb7#9+ no R = A(7)b9sus no b7

16) 1 - 4 - 3 - 4 CΔ7 = Am9 no R = FΔ9#11 no R, 3 = D13sus no R, 5 = B11b9+ no 3, b7

17) 1 - 4 - 4 - 3 CmΔ9 no R = F13b5 no R, 3 = EbΔ7+ = B(7)#9+ no b7 = Bb/11/13/b9 no R, 5

18) 1 - 4 - 5 - 2 Cm/9 = F13 no R, 3 = AbΔ7#11 no R = D11b9 no 3, 5 = Eb6Δ7 no R = B(7)#9b9+ no R, b7 = Gb13b9b5+ no R, 3, b7

19) 1 - 5 - 1 - 5 C13#9 no R, 5 = Gb13#9 no R, 5

20) 1 - 5 - 2 - 4 C7/11 no 5 = FΔ7sus = Dm9+ no R = Gm7/6/11 no R, 5 = Bb/9/#11 no 3 = E(7)b9b5+ no 3, b7

21) 1 - 5 - 3 - 3 C11b9 no R, 5 = Bbm/#11 = E13b9b5 no 3, 5, b7 = Gm7b5/6 no R

22) 1 - 5 - 4 - 2 C11 no R, 5 = E7b9b5 no 3 = Bb/#11 = Dm/9+ = Gm7/6 no R = Db13#9b9 no R, 5, b7 = FΔ7/6sus no 5 = G#°9+ no R, 3 = B°Δ7/11 no R

23) 1 - 6 - 2 - 3 C7#9 no 5 = Gb7/6/#11 no R, 5 = C#mΔ13 no R, 5 = Eb13b9 no 3, b7 = A(7)#9b9#11 no R, 3, b7 = E°Δ7+ no b3 = Bb°9/11 no b3, 6 = G°/11+ no R, b5

24) 1 - 6 - 3 - 2 Cm6/9 no 5 = F7/6 no R = D7b9 no 3 = B7#9b9 no R, 5 = EbΔ7/6/#11 no 3, 5 = A°/11 = F#°7+ no R

25) 1 - 7 - 2 - 2 Cm9 no 5 = D7b9+ no 3 = F7/6sus no R = Bb/9sus no 5 = EbΔ7/6 no 3 = Ab/9/#11 no R

26) 2 - 2 - 2 - 6 C9 no 5 = F#7b5+ no R = D9+ no 3 = Gb7b5+ no R = E7b5+ no 3 = Ab9b5+ no R, b7 = Bb/9/#11 no 5

27) 2 - 2 - 3 - 5 C/9 = Am7/11 no R = D11 no 3, 5 = Em7+ = FΔ13 no R, 3 = Gb7b9b5+ no R,3 = Bb6/9/#11 no R, 5 = G6sus = B(7)#9b9sus+ no R, b7

28) 2 - 2 - 4 - 4 C7+ = F#9b5 no R = Bb9b5 no 3 = D9b5+ no R, 3 = E with #5 & b5 = Ab/9+

29) 2 - 2 - 5 - 3 C6/9 no 5 = D9 no 3 = BbΔ9#11 no R, 5 = Am/11 = Gb7#9b5+ no R, 3 = E7sus+ = B7#9b9sus no R, 5 = Ab(7)b9b5+ no R, b7 = FΔ7/6 no R = G6/9sus no R

30) 2 - 3 - 2 - 5 C6/9 no R = G6/9 no 3 = A7sus = Em7/11 no 5 = FΔ13 no R,5 = D/9sus = BbΔ7/6/#11 no R, 5 = Db(7)#9b9b5+ no R, 3, b7 = F#7#9b9+ no R, 3

31) 2 - 3 - 3 - 4 Cm7b5 = Ab9 no R = Ebm6 = D7b9+ no R = F11b9 no R,3 = Gb6/#11 no 5 = DbΔ13sus no R, 5

32) 2 - 3 - 4 - 3 Cm7 = Eb6 = AbΔ9 no R = F11 no R, 3 = DbΔ13#11 no R,3,5 = Bb6/9sus no 5 = A7#9b9b5 no R, 3 = D11b9+ no R, 3 = G°+/11

33) 2 - 4 - 2 - 4 C7b5 = F#7b5 = D9+ no R = Ab9+ no R = E9b5+ no 3, b7 = Bb9b5+ no 3, b7

34) 2 - 4 - 3 - 3 C7 = F#7b9b5 no R = A7#9b9 no R, 3 = E(7)#9b5+ no 3, b7 = Eb13b9 no R, b7 = Bb6/9/#11 no 3, 5 = FΔ9sus no R

35) 3 - 3 - 3 - 3 C°7 = A°7 = F#°7 = Eb°7 = B7b9 no R = Ab7b9 no R = F7b9 no R = D7b9 no R = Db all four ° extensions = Bb all 4 ° ext. = G all 4 ° ext. = E all 4 ° ext.

Why Are There 43?

As to why there are 43, as I vaguely recall, Ted answered something like, “That’s just nature. That’s just the number you get when you count them all up.” True. But I have not been satisfied with that answer. The number 43 seemed strange to me. So I have investigated where it comes from, mathematically speaking. You can read about my recent research and discoveries in my chapter, *The Mathematics of Four-Note Chords and Beyond*.

—James